GMMap: Memory-Efficient Continuous Occupancy Map Using Gaussian Mixture Model

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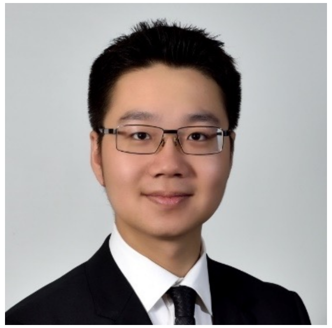
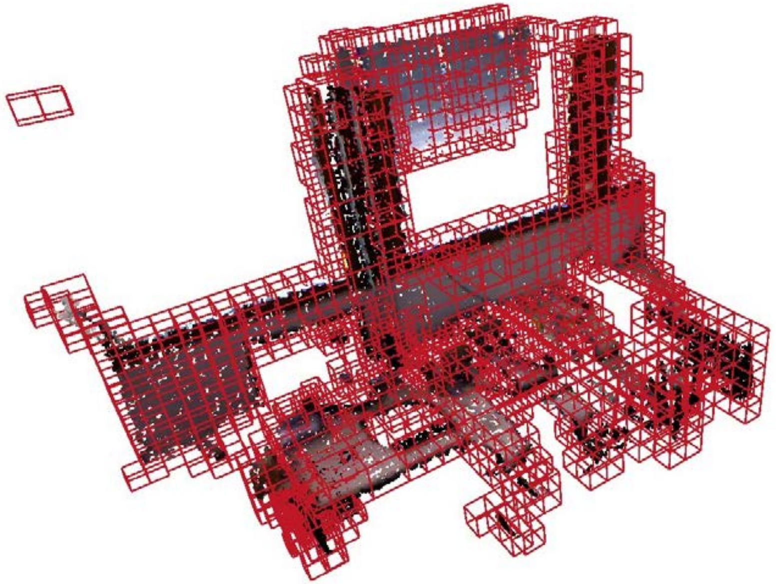


Motivation

Voxel-based 3D Map (Red)



Diverse Applications

Virtual Reality

Autonomous Navigation

Search and Rescue

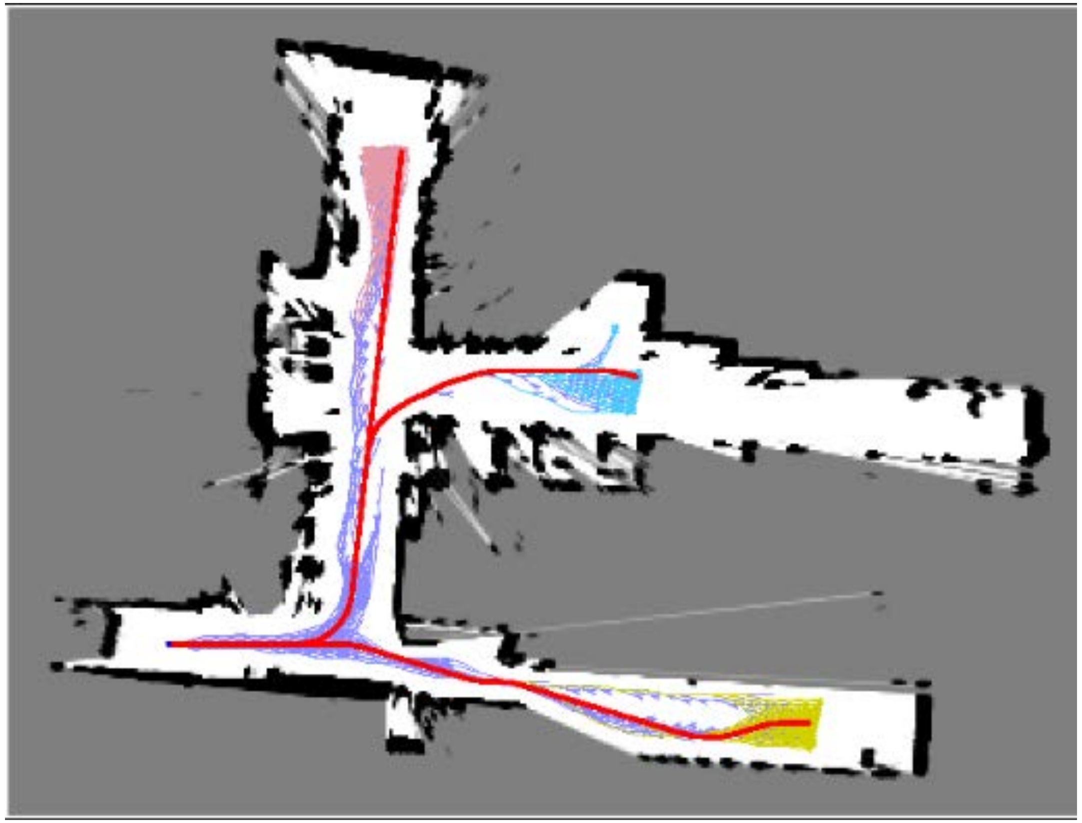
Space Exploration

Robotics Application: Path Planning

- **Black cells:** Contain obstacles
- **White cells:** No obstacles
- **Gray cells:** Unexplored/unknown

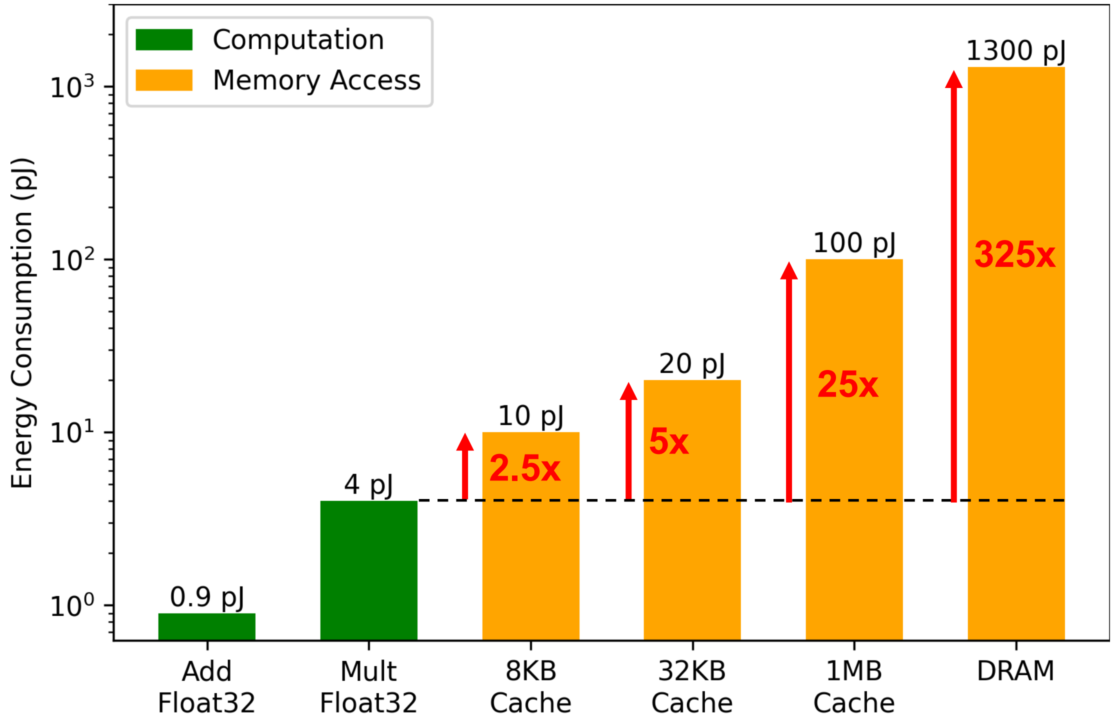
The map should represent both **occupied** (black) and **free** (white) regions to enable safe navigation.

2D Path Planning Example



Importance of Memory Efficiency

- Accurate 3D maps require a large amount of memory to **store** and **construct**



[**Horowitz**, *ISSCC* 2014]

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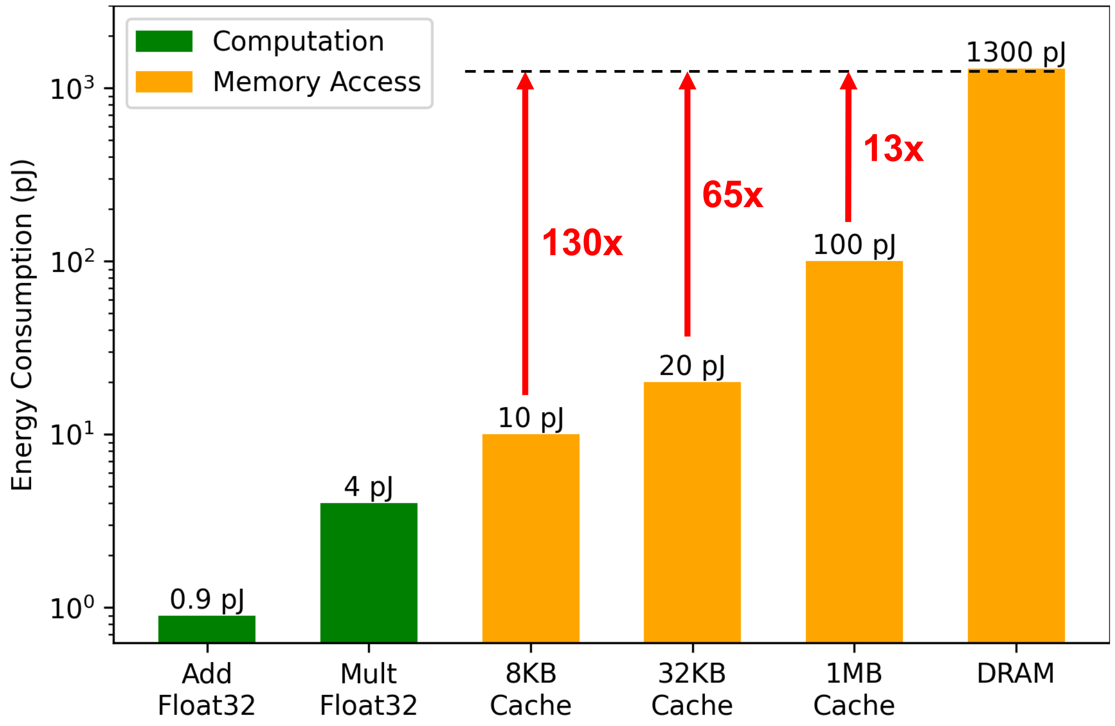


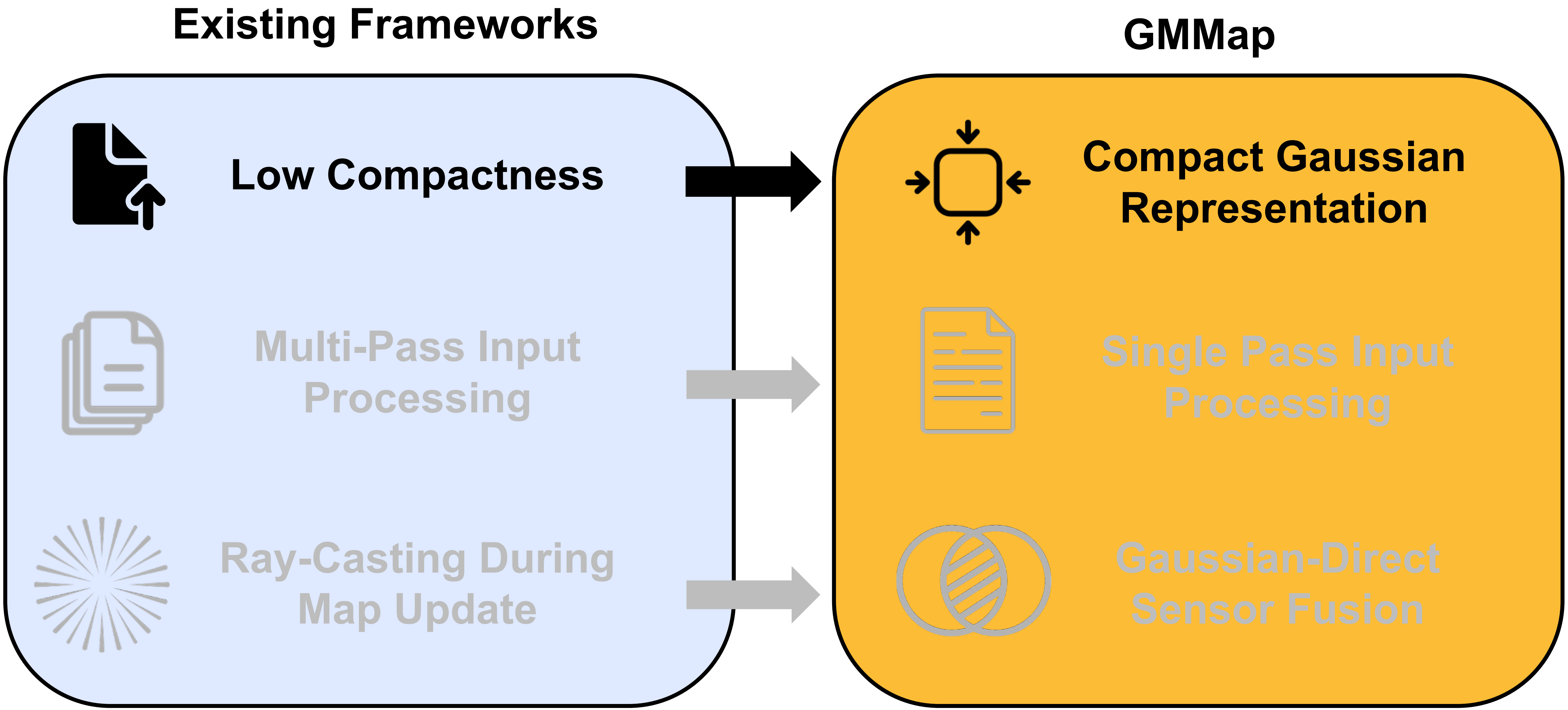
[**Horowitz**, *ISSCC* 2014]

To conserve energy, algorithms should require a **low memory capacity (in KBs)** to encourage cache usage and **reduce DRAM access**!

Summary of Contributions

Existing Frameworks		GMMap
Low Compactness	→	Compact Gaussian Representation
Multi-Pass Input Processing	→	Single Pass Input Processing
Ray-Casting During Map Update	→	Gaussian-Direct Sensor Fusion

Summary of Contributions



Existing Frameworks Are Not Memory Efficient

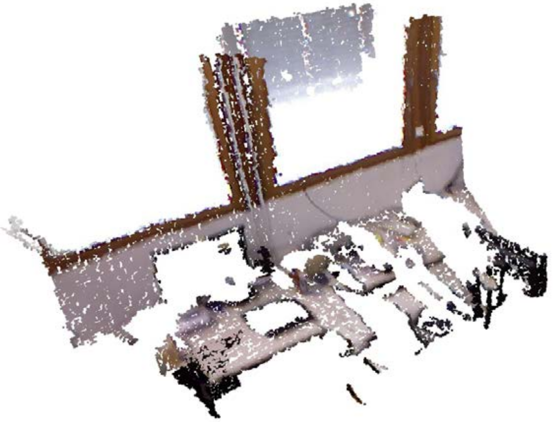
1. Low compactness

– Voxel-based representations requires 10’s MBs to GBs to store (for thousands of depth images)

2D Depth Image

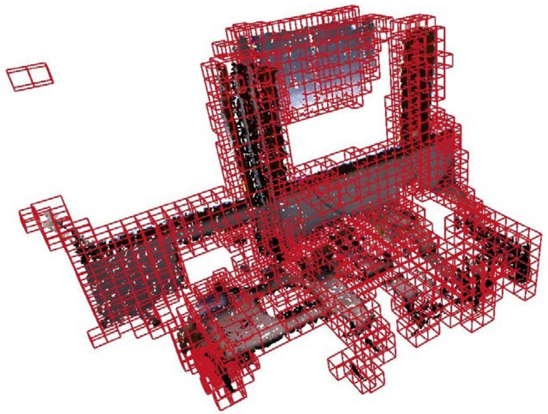
1.2MB

Point Cloud



2MB

Voxel-based 3D map (red)



147KB

Existing Frameworks Are Not Memory Efficient

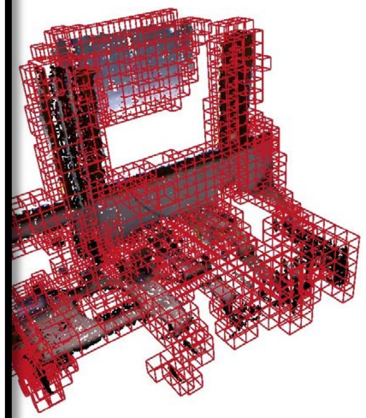
1. Low compa

– Voxel-based … f depth images)

2D Depth Im … **sed 3D map (red)**

1.2MB

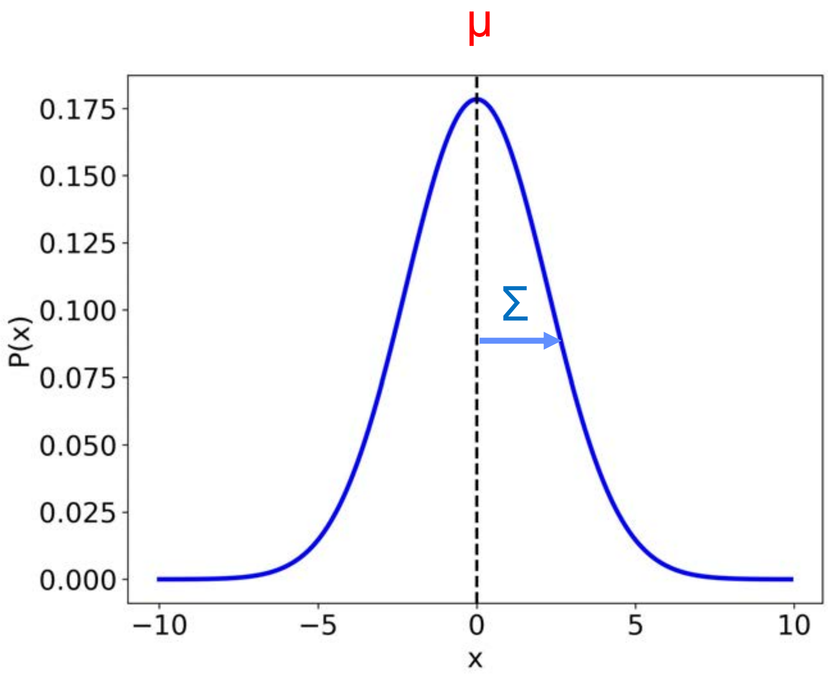
10.5MB



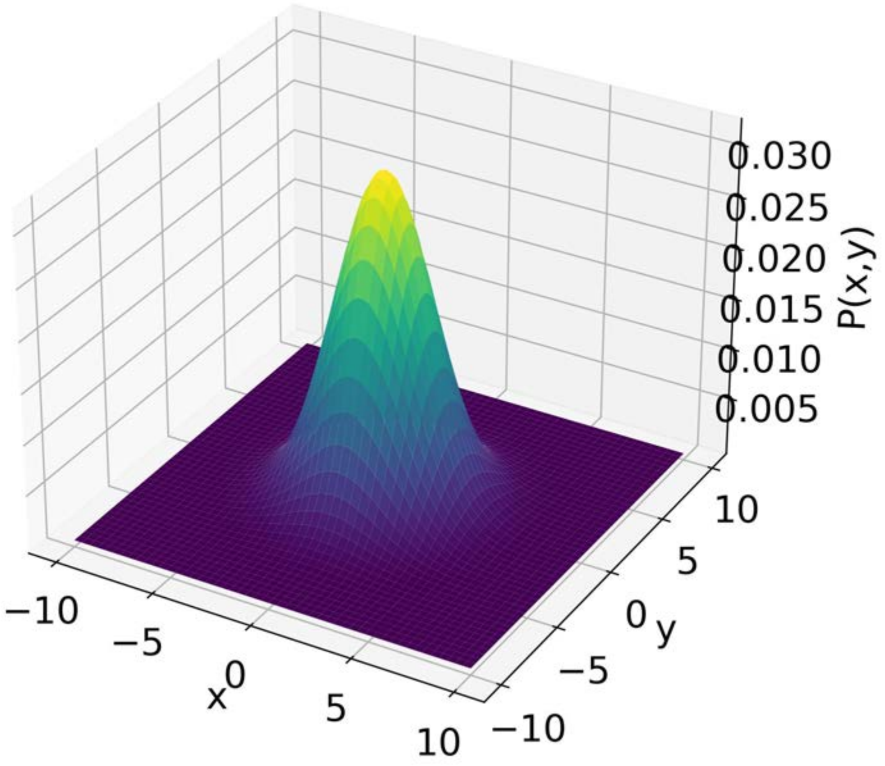
147KB

What is a Gaussian distribution?

- Probabilistic distribution parametrized by **mean (μ)** and **covariance matrix (Σ)**



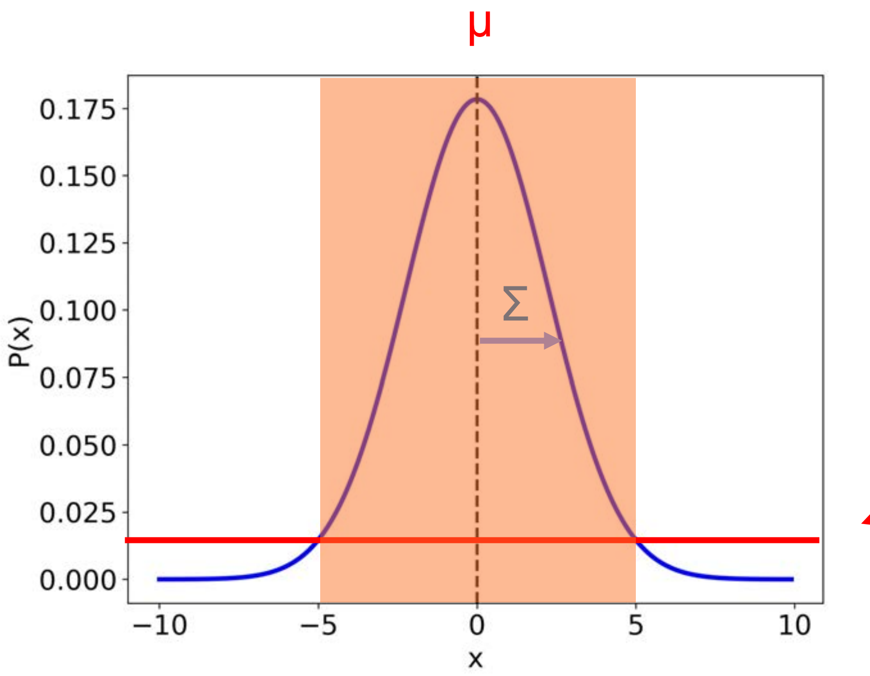
1D Gaussian



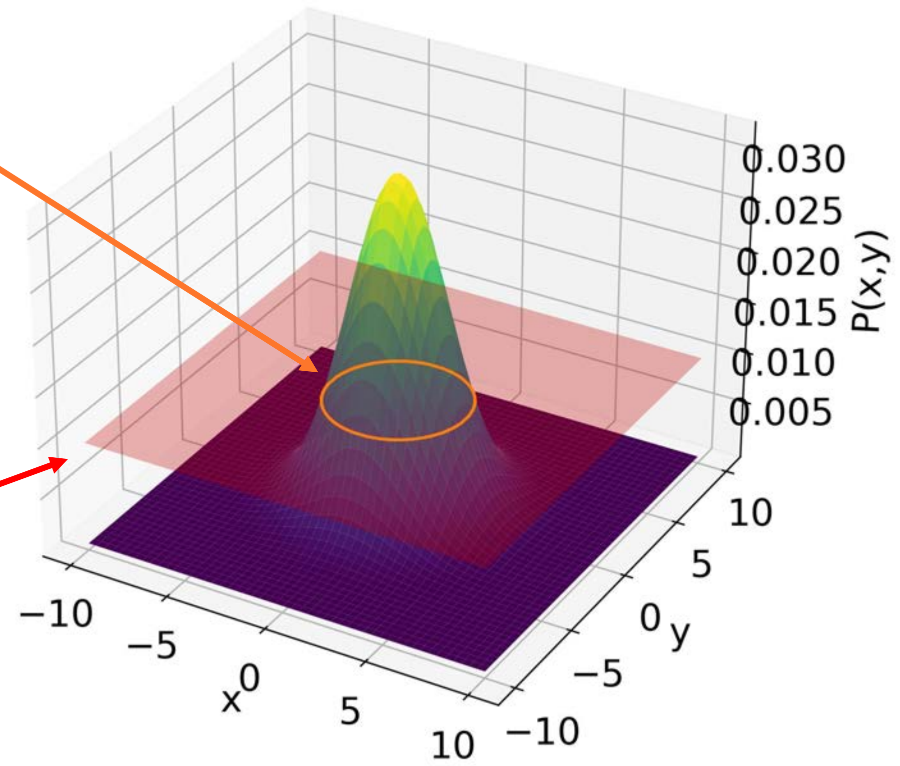
2D Gaussian

What is a Gaussian distribution?

- Probabilistic distribution parametrized by **mean (μ)** and **covariance matrix (Σ)**
- Iso-surface is visualized as an **ellipsoid**



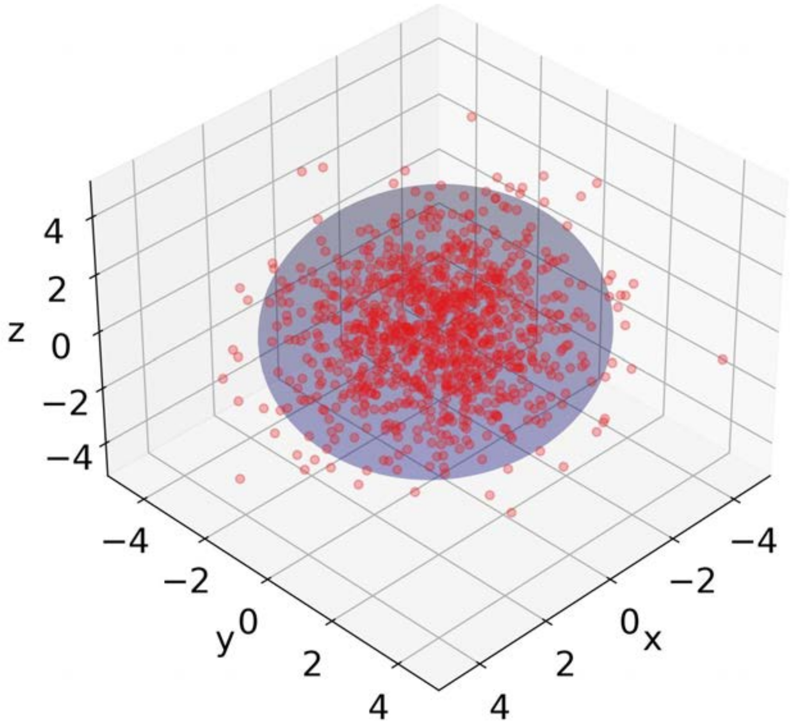
1D Gaussian



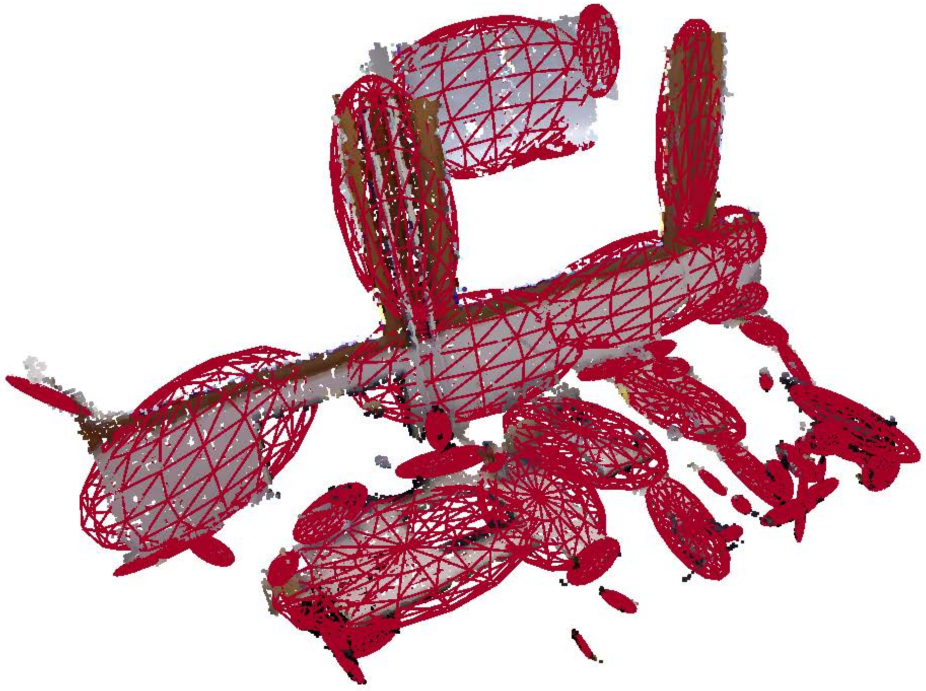
2D Gaussian

3D Gaussian Distribution

- Mean (3 parameters) and covariance (6 parameters) matrix
- Iso-surface encloses the point cloud of objects in 3D space



1 Gaussian



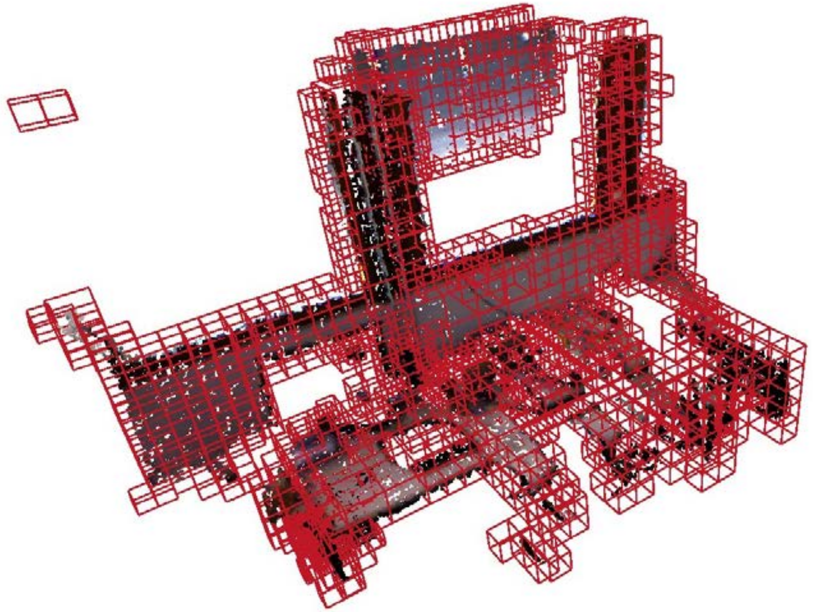
54 Gaussians (Red)

Compact Map Representation Using Gaussians

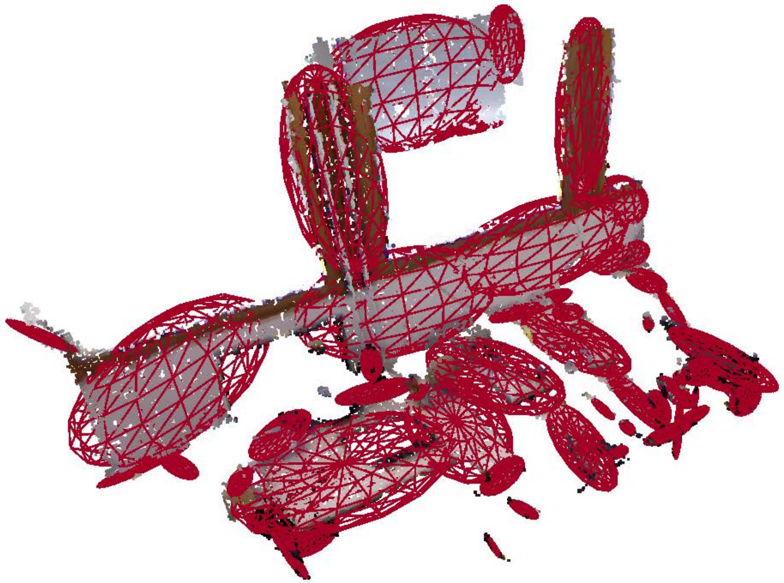
Gaussian map representations are highly compact

– Visualized as ellipsoids in 3D. Highly flexible representation.

Voxel-based 3D map (red)



GMMap (red)



147KB → **16x reduction** → 9KB

Compact Map Representation Using Gaussians

Gaussian map representations are highly compact

– Visualized as ellipsoids in 3D. Highly flexible representation.

Voxel-based 3D map (red)

10.5MB

GMMap (red)

840KB

13x reduction →

Summary of Contributions

Existing Frameworks		GMMap
Low Compactness	→	Compact Gaussian Representation
Multi-Pass Input Processing	→	**Single Pass Input Processing**
Ray-Casting During Map Update	→	Gaussian-Direct Sensor Fusion

Existing Frameworks Are Not Memory Efficient

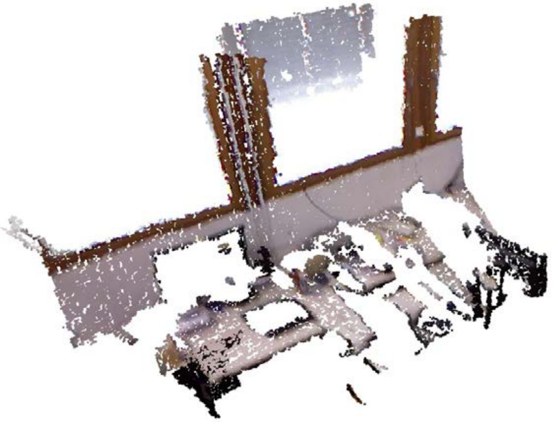
2. Multi-pass processing of input data

– Determining the parameters of Gaussians requires **multi-pass** processing

2D Depth Image

1.2MB

Point Cloud



2MB

GMMap (red)

9KB

Existing Frameworks Are Not Memory Efficient

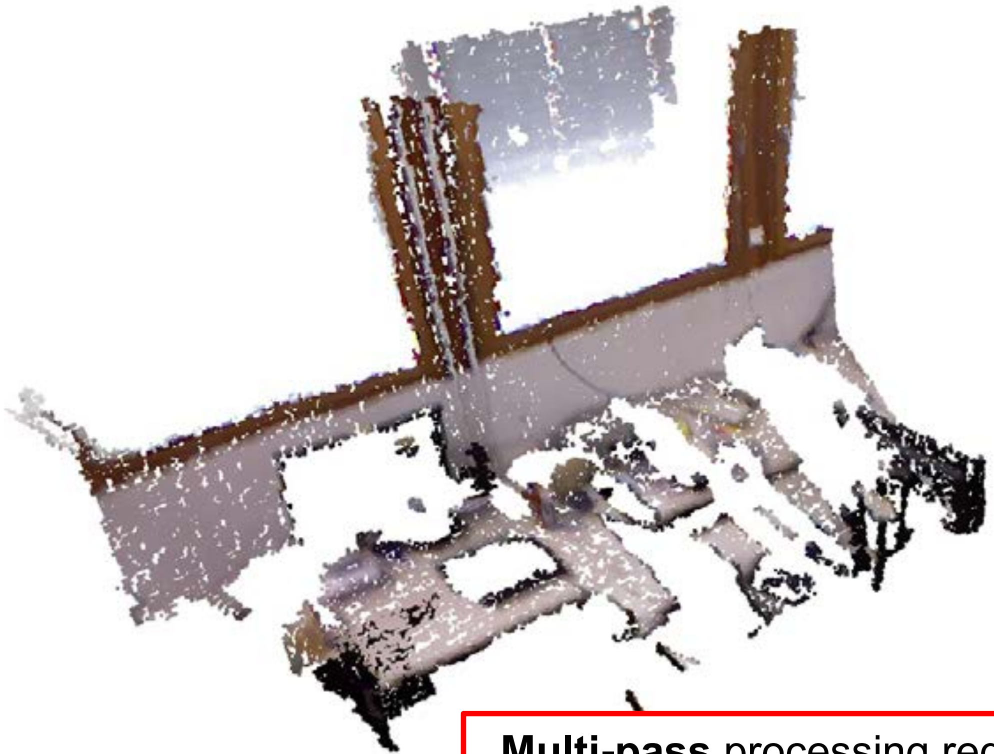
- Points (measurements) are **near to each other** on the **same planar surface** are represented by Gaussians.

Gaussians

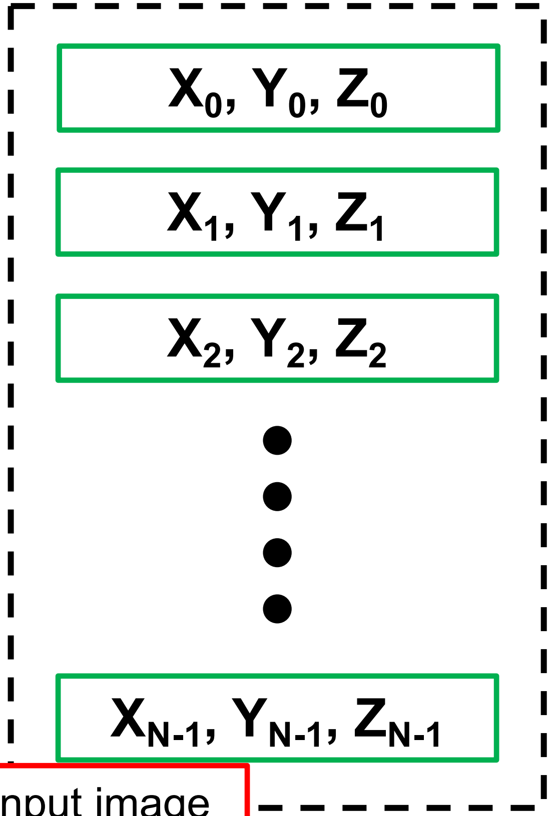
Existing Frameworks Are Not Memory Efficient

- Point cloud (a list of **unorganized** 3D points) does not encode any spatial relationships among the points.

Point Cloud



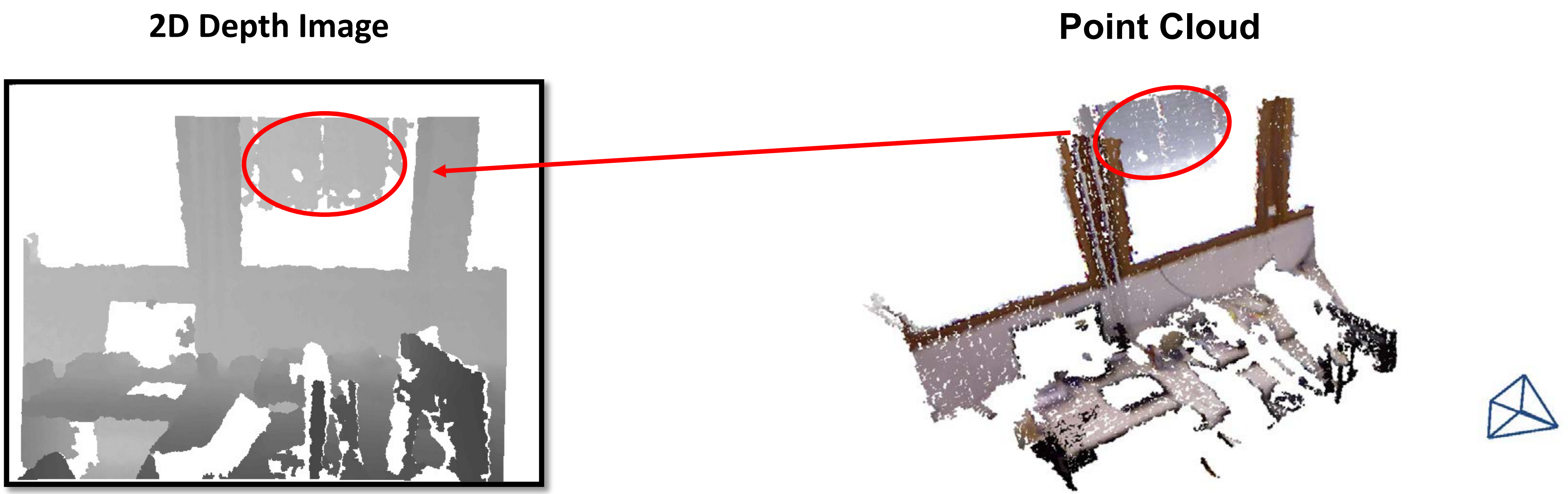
List of 3D Points



Multi-pass processing requires the storage of input image and/or point cloud (1.2MB to 2MB of memory **overhead**).

Single Pass Gaussian Fitting (SPGF)

- Depth image encodes spatial relationship among measurements
- Measurements that are **neighbors in 3D** are also likely **neighbors in 2D** depth image.

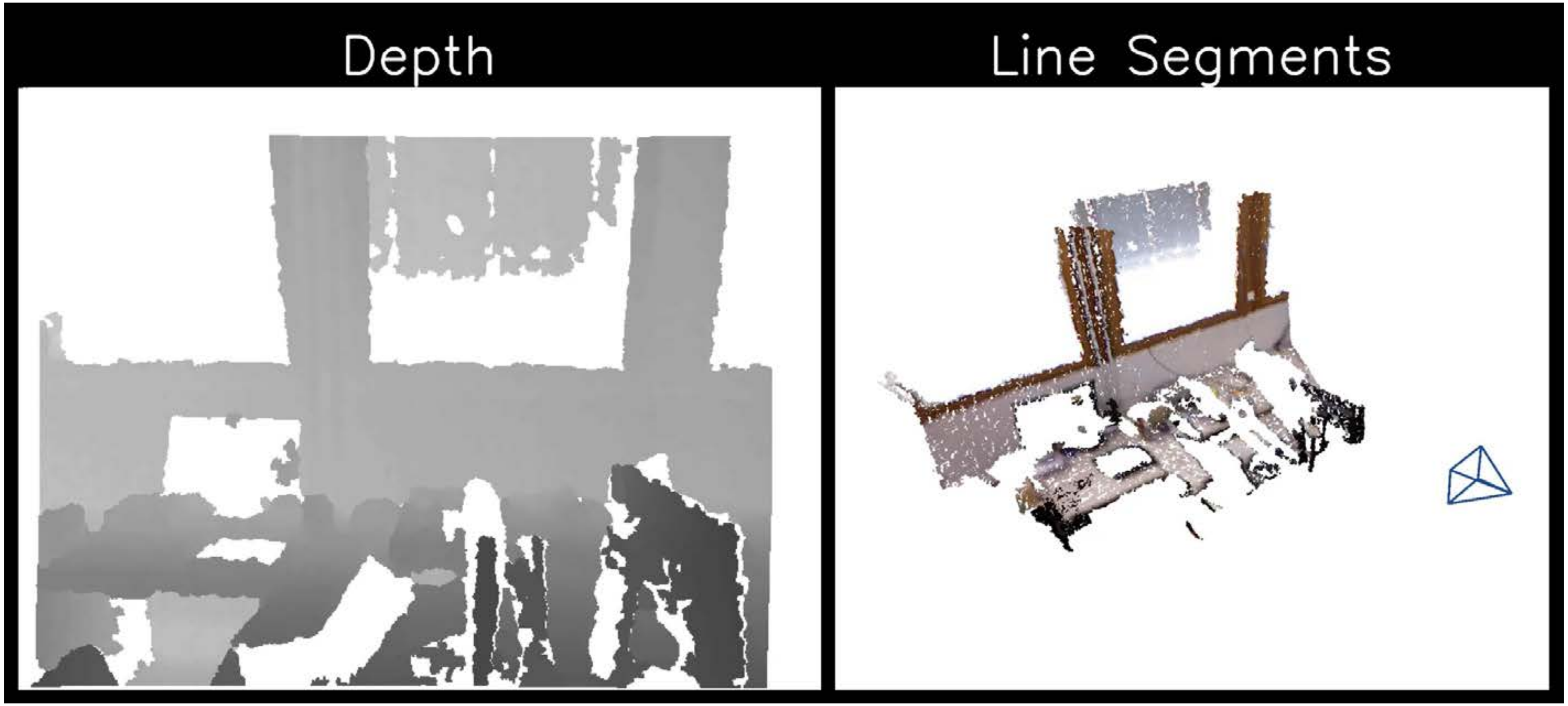


Connectivity of 3D surfaces is efficiently inferred by processing **neighboring pixels** in depth image!

Single Pass Gaussian Fitting (SPGF)

- **Two Steps:**
  1. **Scanline Segmentation:** Inferring connectivity horizontally to create line segments in 3D

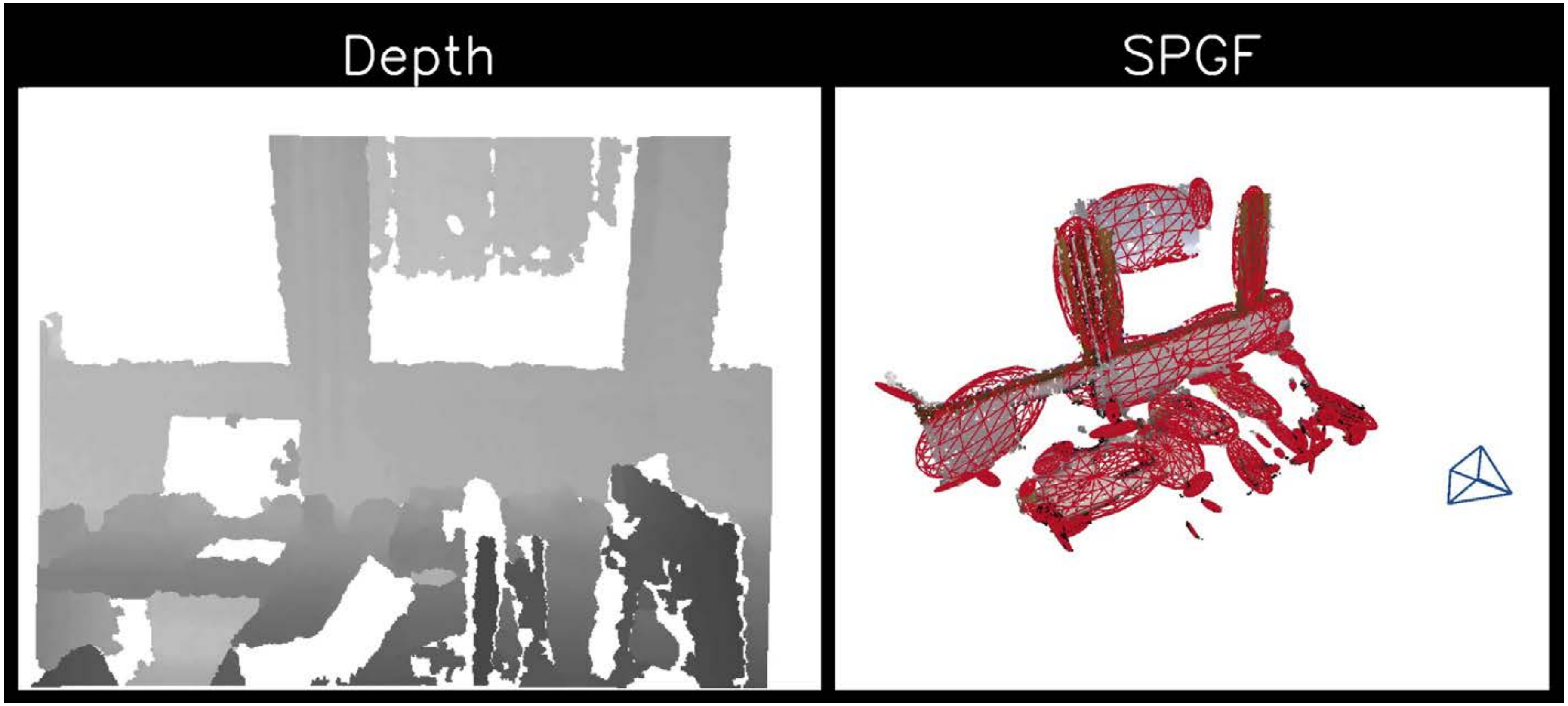
Row-by-row image streaming



Single Pass Gaussian Fitting (SPGF)

- **Two Steps:**
  1. **Scanline Segmentation:** Inferring connectivity horizontally to create line segments in 3D
  2. **Segment Fusion:** Inferring connectivity vertically to fuse line segments into Gaussians

Row-by-row image streaming



Single Pass Gaussian Fitting (SPGF)

- Single-pass row-by-row based processing in raster-scan order.
- Only one pixel is required in memory at anytime!

2D Depth Image

Single-pass Processing →

Gaussian 3D map (red)

At similar map accuracy, SPGF requires only **43KB of overhead** (input and temporary variables) which is at least **88% lower** than prior multi-pass approaches!

Summary of Contributions

Existing Frameworks		GMMap
Low Compactness	→	Compact Gaussian Representation
Multi-Pass Input Processing	→	Single Pass Input Processing
Ray-Casting During Map Update	→	**Gaussian-Direct Sensor Fusion**

Sensor Fusion Across Multiple Depth Images

- Sensor measurements for the **same object** need to be **fused** to save memory

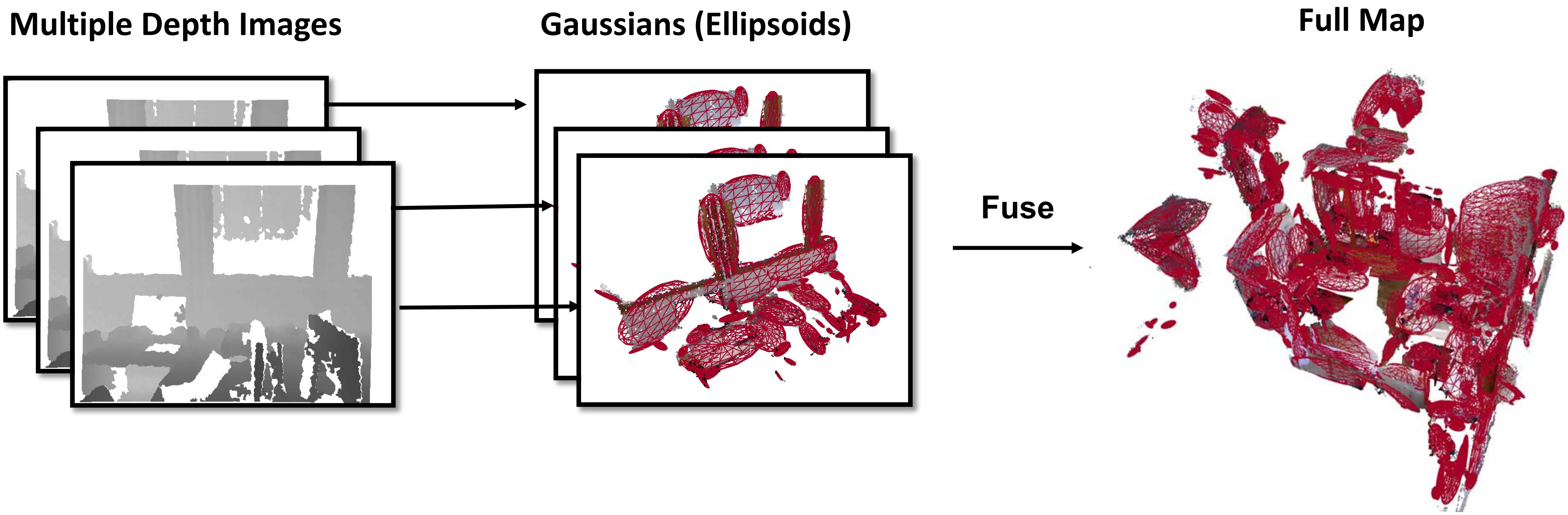
Depth Image

Single Pass Processing →

Gaussians (Ellipsoids)

Sensor Fusion Across Multiple Depth Images

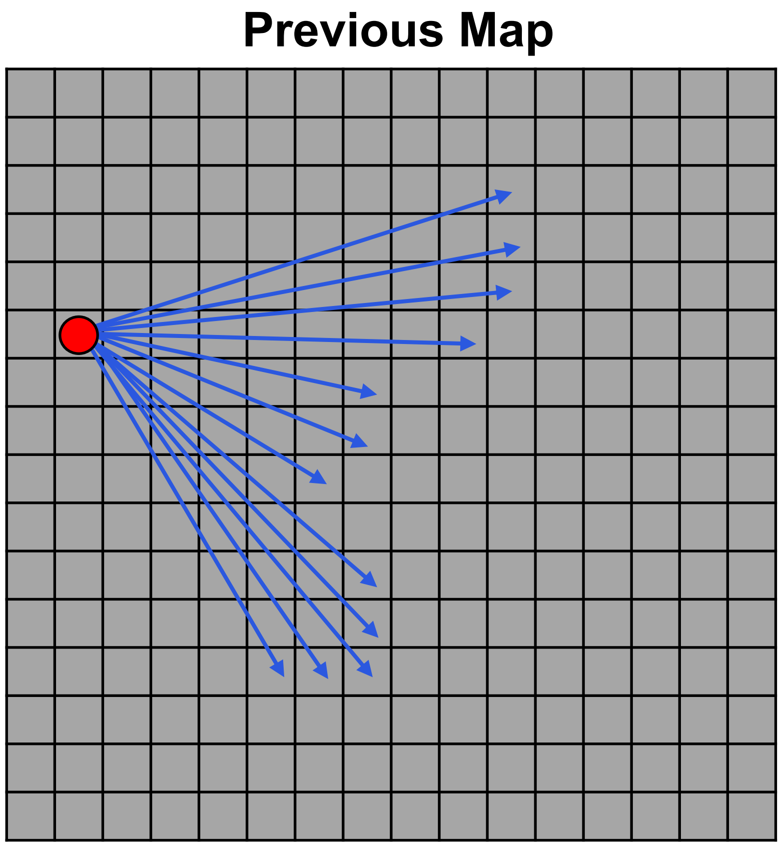
- Sensor measurements for the **same object** need to be **fused** to save memory



Existing Frameworks Are Not Memory Efficient

3. Ray-casting required for map update

– Memory accesses along the rays often **lack spatial and temporal locality**



Existing Frameworks Are Not Memory Efficient

3. Ray-casting required for map update

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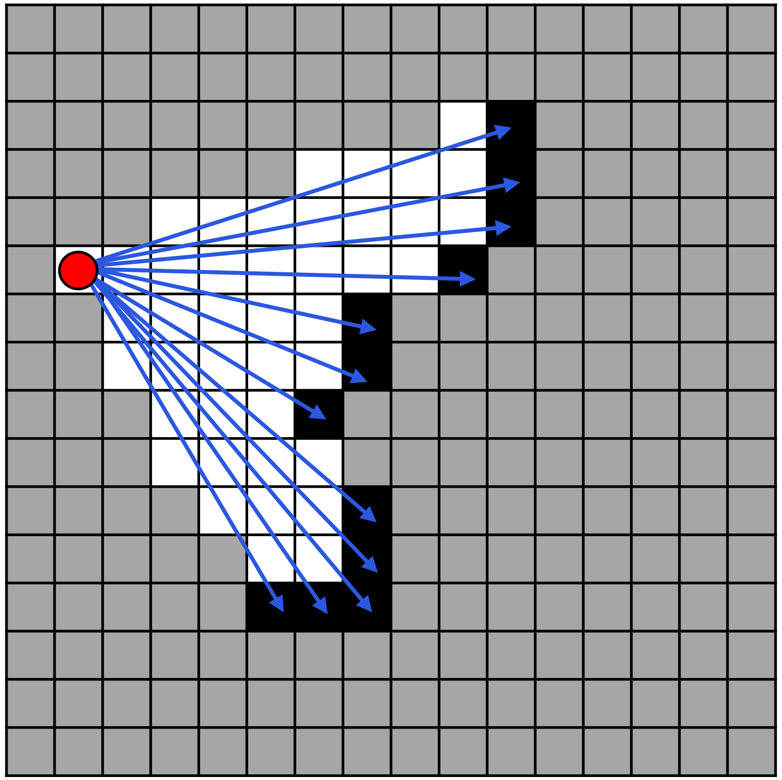
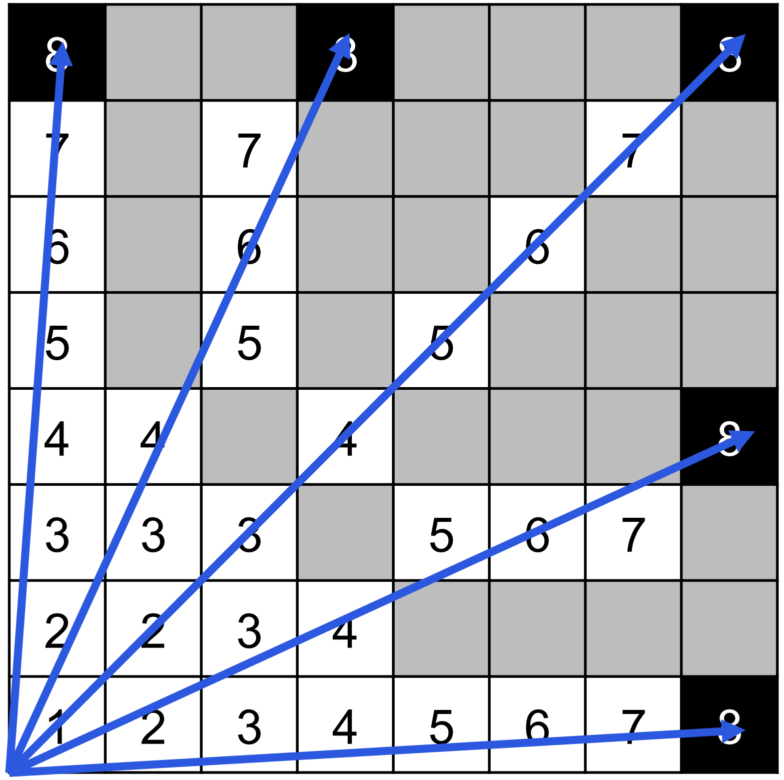
Updated Map



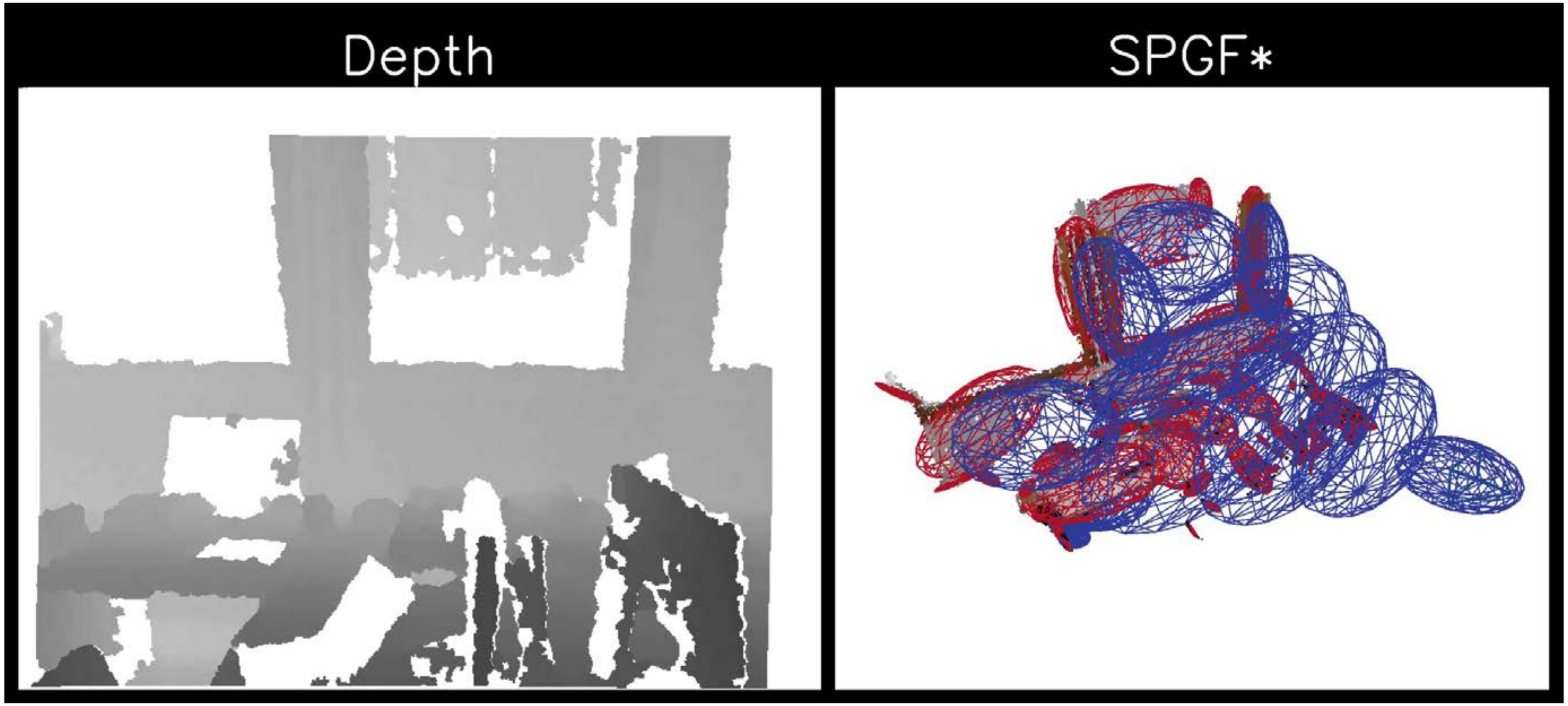
Memory Access Pattern



SPGF*: Free Space Extension

- Region *along the sensor rays* are obstacle-free
- Gaussians in free space (blue) are also created in a single pass!

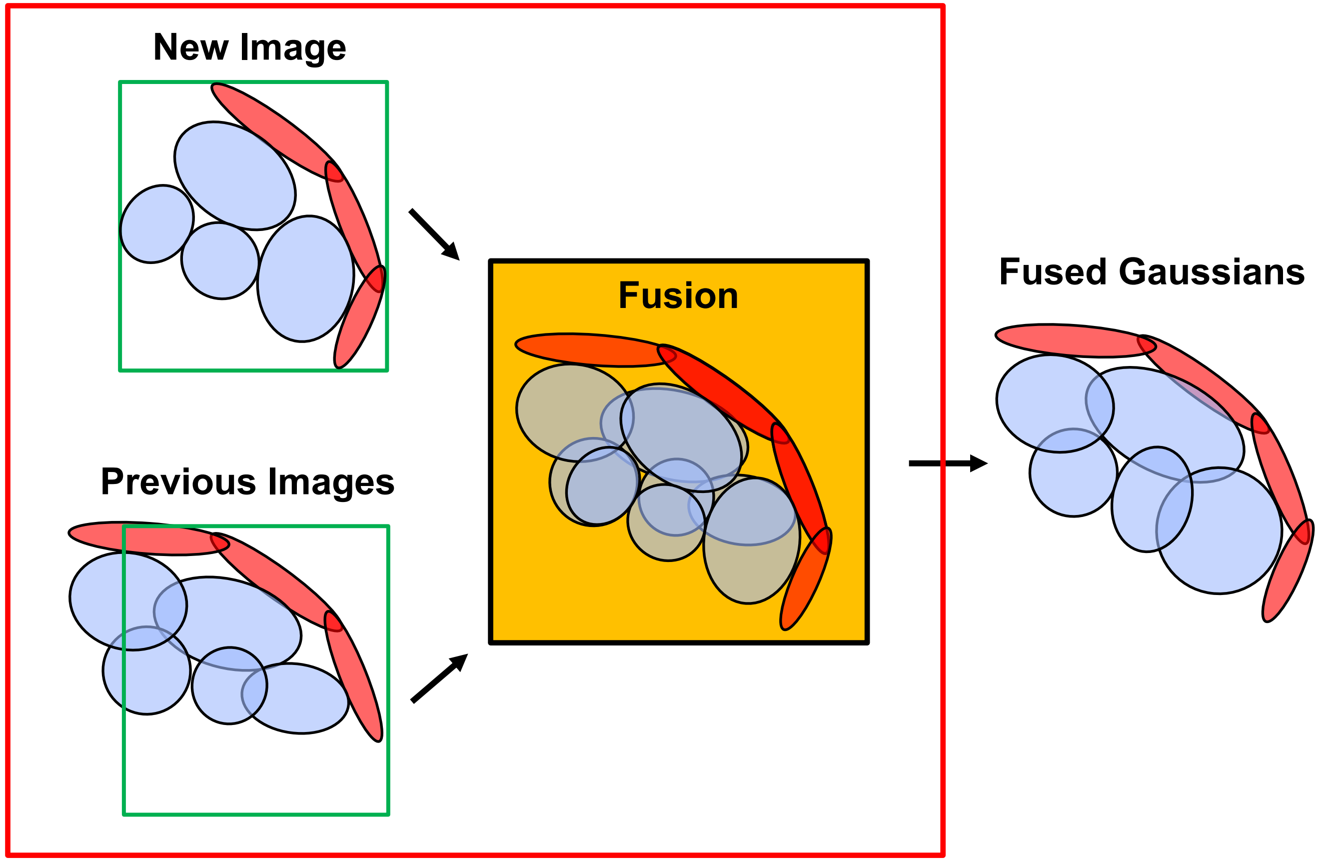
Row-by-row image streaming



Gaussian-Direct Sensor Fusion

- Sensor measurements for the **same object** need to be **fused** to save memory
- Gaussians can be directly fused to update the map *without ray casting*

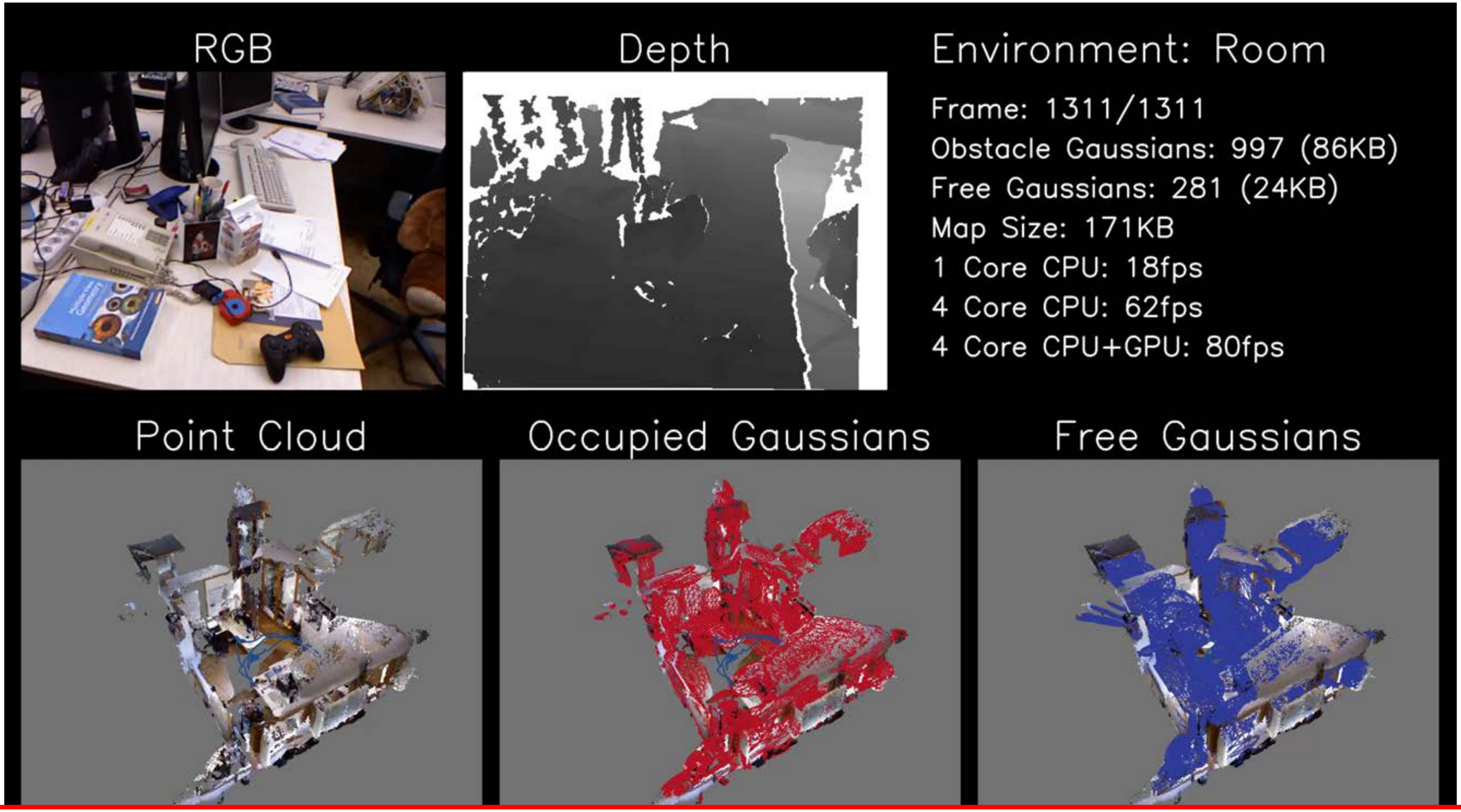
Cache Friendly & More Compute Efficient



On a **low-power ARM CPU**, the DRAM access and cache miss rate are reduced by around **78%** compared with prior works that require ray casting.

Gaussian-Direct Sensor Fusion

- Low-power ARM Cortex A57 CPU with 4 cores + Pascal GPU with two SMs



Using the CPU, GMMap enables real-time map construction with **3.6x to 116x** higher throughput, **69% to 98%** lower energy compared with prior works.

Accuracy of GMMap

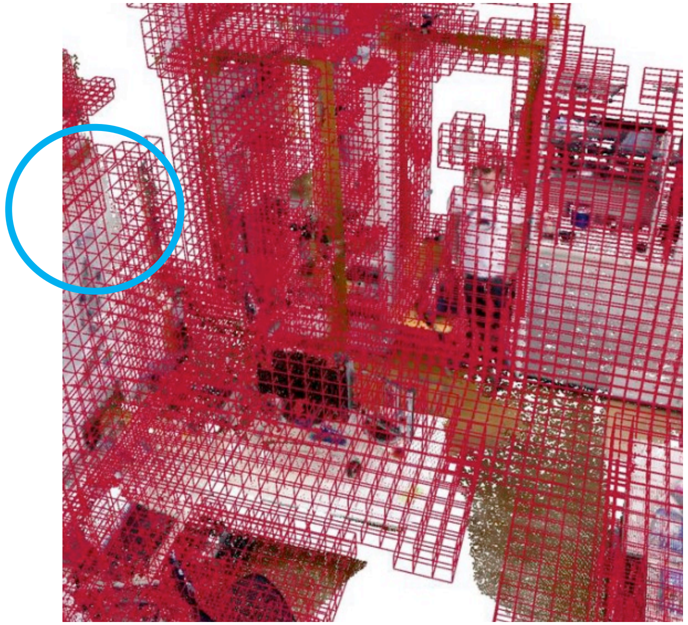
- **Indoor *Room* Environment**

GMMap

Occupied Gaussians

Uniform Sampling →

Occupied Voxels



Each Gaussian has no bound and **extends beyond its ellipsoidal wireframe**.
Obstacles that are not covered by ellipsoids are in fact preserved.

Accuracy of GMMap

- **Indoor *Room* Environment**

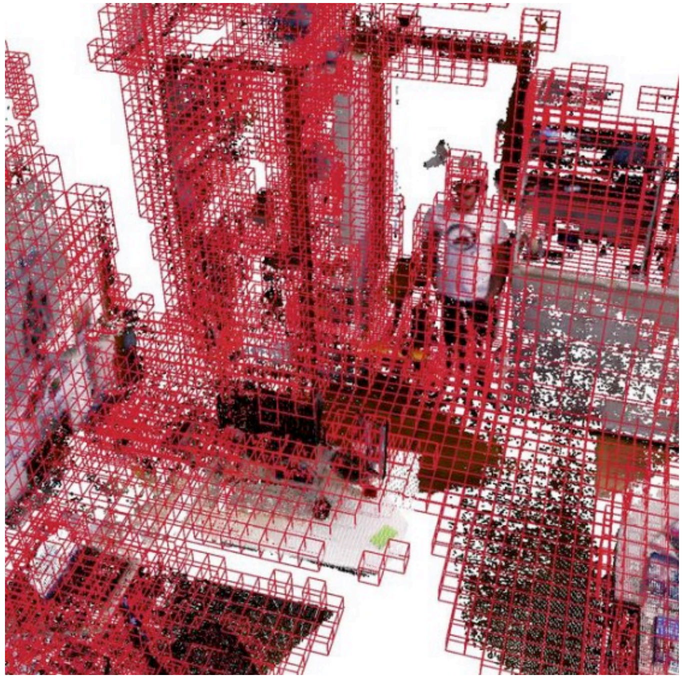
GMMap

Occupied Gaussians

Map Size: 176 KB
Accuracy: 96%

OctoMap [Hornung et al.]

Occupied Voxels



Map Size: 2,190 KB
Accuracy: 93%

Accuracy of GMMap

- **Indoor *Room* Environment**

GMMap

Occupied Gaussians

Map Size: 176 KB
Accuracy: 96%

NDT-OM [Saarinen et al.]

Occupied Gaussians

Map Size: 426 KB
Accuracy: 93%

Across indoor and outdoor environments, GMMap achieves **comparable accuracy** while reducing the map size by **56% to 98%** compared with prior works.

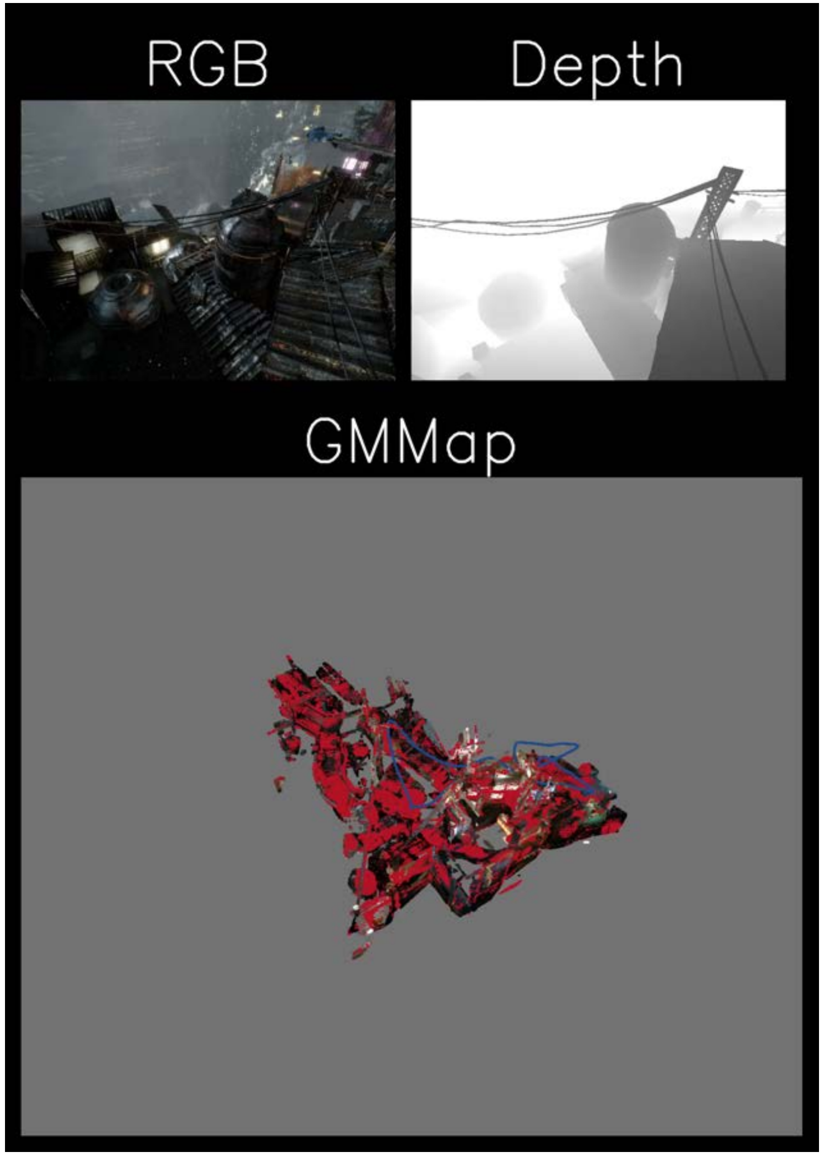
Summary

- **GMMap enables memory-efficient mapping by:**
    1. Compact Gaussian representation
    2. Single-pass input processing
    3. Gaussian-direct sensor fusion
- **Comparable accuracy as prior works with state-of-the-art compactness, throughput and energy consumption**.

P. Z. X. Li, S. Karaman, V. Sze, "Memory-Efficient Gaussian Fitting for Depth Images in Real Time," IEEE International Conference on Robotics and Automation (ICRA), May 2022

P. Z. X. Li, S. Karaman, V. Sze, "GMMap: Memory-Efficient Continuous Occupancy Map Using Gaussian Mixture Model," arXiv, June 2023

Specialized hardware acceleration of the GMMap is coming soon!



Sponsors: NSF RTML 1937501, NSF CPS 1837212